10 Many variations on the sine and cosine functions are also periodic functions.

a. Find the amplitude, period, and y-displacement for each of the following functions.

i. $y = 2\cos(-x) + 3$

amplitude = 2

period = 2π

y-displacement (or midline) = 3

ii. $y = -3\sin 0.1x + 5$

amplitude = 3

period = $\frac{2\pi}{0.1} = 20\pi$

y-displacement (or midline) = 5

iii. $y = 12\sin 3x - 8$

amplitude = 12

period = $\frac{2\pi}{3}$

y-displacement (or midline) = -8

b. Suppose data from a periodic variable are fit well by a function of the form $y = a\cos bx + c$ where $a < 0$ and $b > 0$. A plot of the data suggests a y-displacement of -5, amplitude of 7, and period of 6π. Write a function rule for this data.

period = $\frac{2\pi}{b}$

$6\pi = \frac{2\pi}{b}$

$b = \frac{2\pi}{6\pi} = \frac{1}{3}$

$y = -7\cos\frac{1}{3}x - 5$

12 Suppose that you are trying to model the motion of a clock pendulum that moves as far as 5 inches to the right of vertical and swings with a period of 2 seconds.

a. Find variations of $d(t) = \cos t$ that fit the conditions for each part below.

i. A modeling function whose values range from -5 to 5 and has a period of 2π

$d(t) = 5 \cos t$

ii. A modeling function that has a period of 2 and whose values range from -1 to 1

$d(t) = \cos \pi t$

$\text{period} = \frac{2\pi}{b}$

$2 = \frac{2\pi}{b}$

$b = \frac{2\pi}{2} = \pi$

iii. A modeling function that has a period of 2 and whose values range from -5 to 5

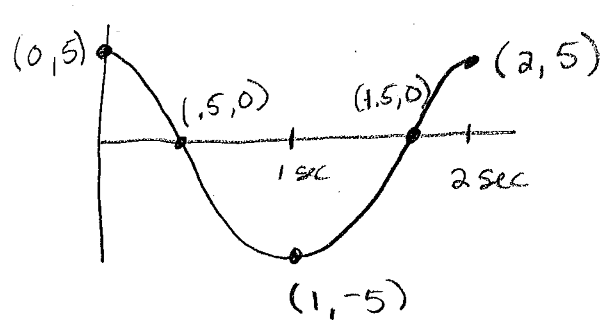
$d(t) = 5 \cos \pi t$

b. How are the numbers in the function for Part aiii related to the motion of the pendulum you are modeling?

Since the pendulum swings 5 inches to the right from the vertical, the starting value is 5. To have a two second period, the value of b must be π.

c. Graph the function that models the motion of the clock pendulum. Identify the coordinates of the t-intercepts and minimum and maximum points of the graph.

Graph aiii:



19 Use the language of geometric transformations to describe the relationships between the graphs of the following pairs of functions.

a. $f(x) = -\cos t$ and $g(x) = \cos t$

f(x) is a reflection over the x-axis of g(x)

b. $h(x) = 5 + \sin t$ and $k(x) = \sin t$

h(x) is a vertical translation of 5 units of k(x)

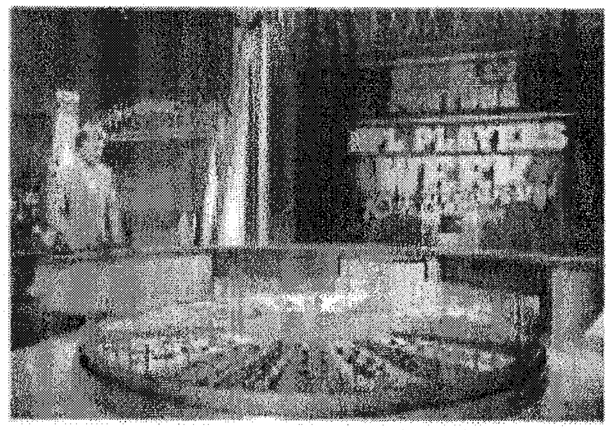
c. $m(x) = 5 \cos t$ and $n(x) = \cos t$

m(x) is a vertical stretch of 5 of n(x)

41 Recall that angles can be measured in both degrees and radians, with the two measurement scales related by the fact that 180° = π radians or 360° = 2π radians. Complete a copy of the following table showing degree and radian equivalents for some important angles.

Degrees	0	30	45	60	90	120	135	150	180
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π

3 *Wheel of Fortune* has been the most popular game show in the history of television. On the show, three contestants take turns spinning a large wheel similar to the one at the right. The result determines how much that contestant wins as she or he progresses toward solving a word, phrase, or name puzzle. The wheel is divided into 24 sectors of equal size, each corresponding to a dollar value or some other outcome.



a. If the wheel spins through 3.8 counterclockwise revolutions, what is the degree measure of the angle through which the wheel spins?

$3.8(360) = 1368°$

Imagine a coordinate system superimposed on the wheel of fortune with its origin at the center of the wheel. Suppose a contestant spins the wheel releasing it at the "3 o'clock" position on the coordinate system, and the wheel completes 2.8 revolutions in 20 seconds.

d. What is the angular velocity in revolutions per second? In radians per second?

$\frac{2.8}{20} = .14$ rpsecond

$\frac{.14 \text{ rev}}{1 \text{ sec}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rev}} = .28\pi$ rad/sec or $\approx .88$ rad/sec

e. If the wheel is 10 feet in diameter, what is the average linear velocity in feet per second of a point on the edge of the wheel?

Linear velocity $= .14(10\pi) = 1.4\pi$ ft/sec

or

≈ 4.4 ft/sec

f. What are the coordinates of the release point when the wheel stops at the end of 20 seconds? Explain your reasoning.

2.8 revolutions is $2.8(2\pi)$ radians or 5.6π rad

$(10\cos 5.6\pi, 10\sin 5.6\pi)$

$(3.09, -0.95)$

4 The Thames River passes through the heart of London, England. Ships entering that part of the river need to pass under a number of bridges like the famous Tower Bridge. The height of the Tower Bridge above the river (in meters) varies over time (in hours following high tide) according to $d(t) = 12 + 3.4 \cos 0.5t$.

a. What are the maximum and minimum distances from the bridge to the river?

maximum: $12 + 3.4 = 15.4$ m

minimum: $12 - 3.4 = 8.6$ m

b. What are the period and amplitude of variation in distance from the bridge to the river?

period $= \frac{2\pi}{0.5} = 4\pi$ hours

amplitude $= 3.4$ m

c. How often in one day does the distance from bridge to river complete a full cycle from maximum to minimum and back to maximum?

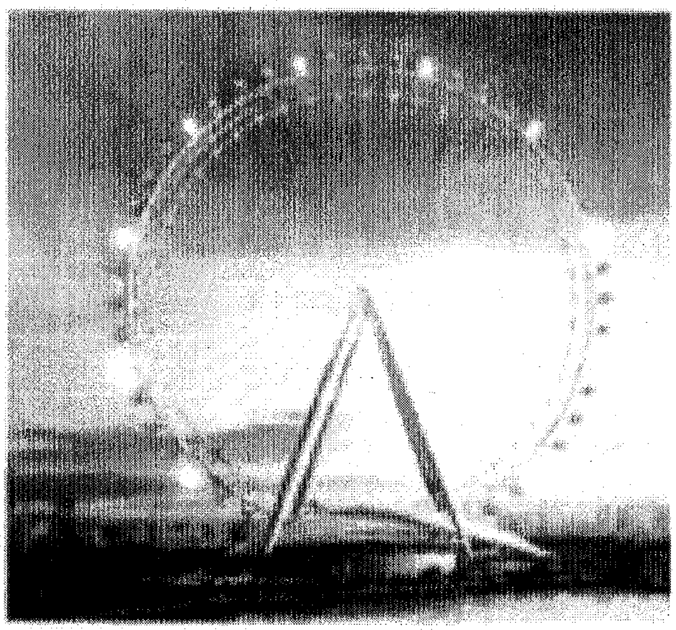
If period is 4π hours, then frequency is $\frac{1}{4\pi}$.

However, 1 day has 24 hours so

$\frac{24}{4\pi} = 1.9$

Most tidal waters "advance" about an hour each day.

35 As part of the preparation for the 2008 Beijing Olympics, a great observation wheel with a diameter of 198 meters was to be built. The wheel has 48 capsules which are evenly spaced around the outside of the wheel.



a. What is the distance from one capsule to the next along the outside of the wheel?

$C = 198\pi$ $\qquad \frac{198\pi}{48} = 12.96m$

c. If the wheel completes one revolution every 30 minutes, what is the linear velocity of the wheel in kilometers per hour?

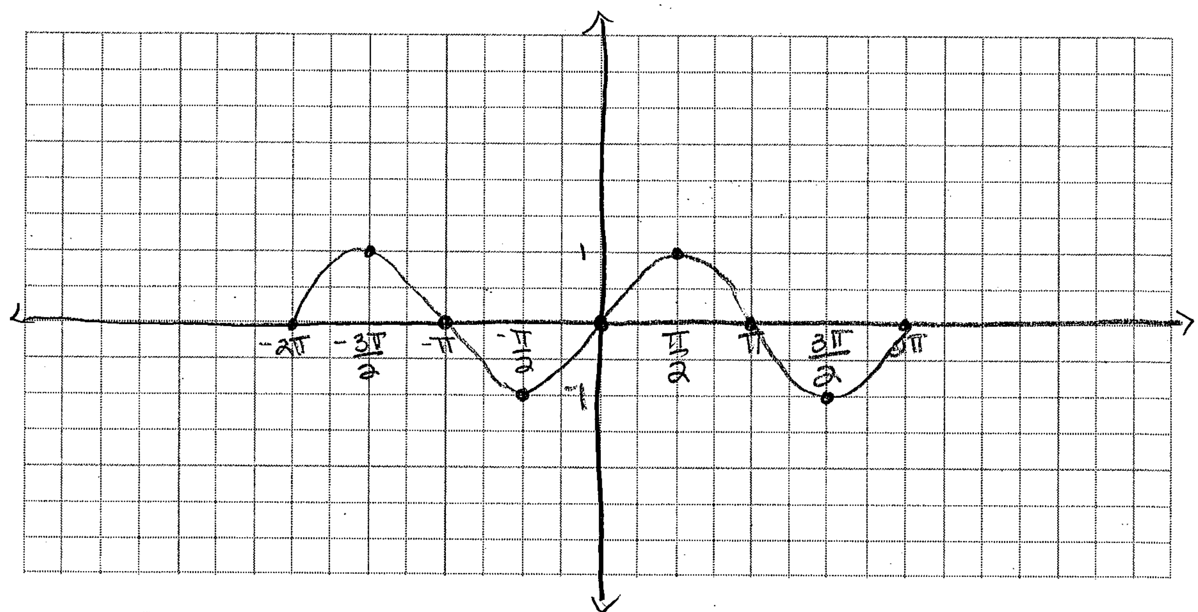
linear velocity = ang vel · circumference

$= \frac{1 \text{ rev}}{30 \text{min}} \cdot 198\pi \text{ m} \cdot \frac{60 \text{min}}{1 \text{hr}} \cdot \frac{1 \text{km}}{1000 \text{m}}$

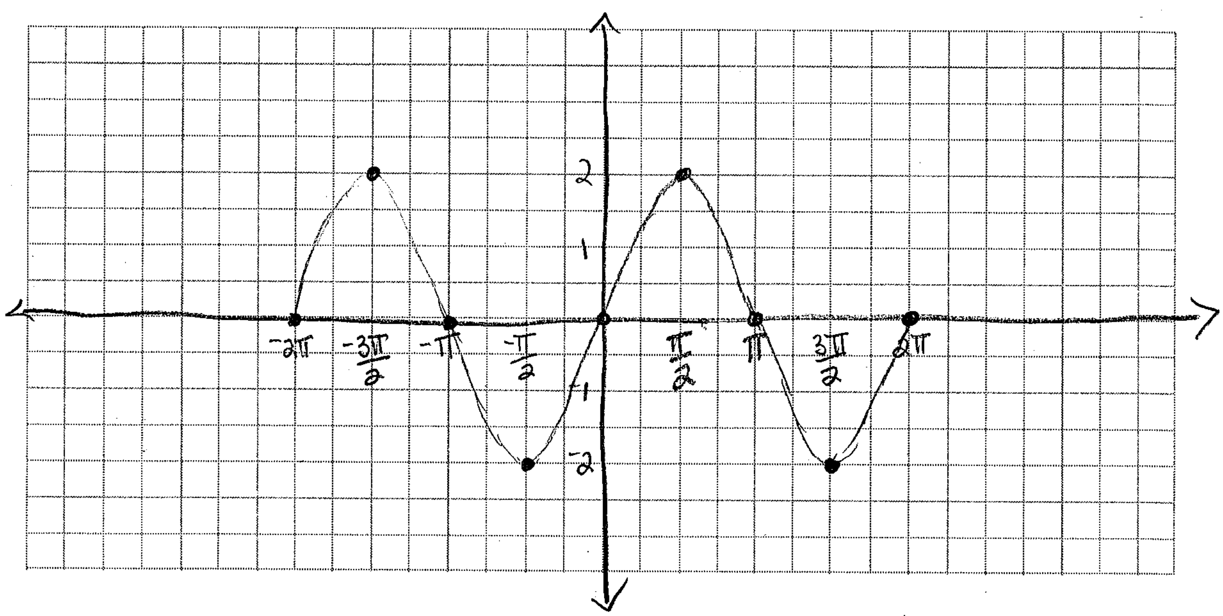
$= 1.244 \text{ km/hr}$

43 Without using technology, sketch a graph of each function over the interval $[-2\pi, 2\pi]$. Then give the period and the amplitude of each function. Use technology to check your work.

a. $y = \sin x$



b. $y = 2 \sin x$



c. $y = \sin 2x$